Phantom energy from graded algebras

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We construct a model of phantom energy using the graded Lie algebra SU(2/1). The negative kinetic energy of the phantom field emerges naturally from the graded Lie algebra, resulting in an equation of state with $w < -1$. The model also contains ordinary scalar fields and anti-commuting (Grassmann) vector fields which can be taken as two component dark matter. A potential term is generated for both the phantom fields and the ordinary scalar fields via a postulated condensate of the Grassmann vector fields. Since the phantom energy and dark matter arise from the same Lagrangian the phantom energy and dark matter of this model are coupled via the Grassman vector fields. In the model presented here phantom energy and dark matter come from a gauge principle rather than being introduced in an *ad hoc* manner.

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I. INTRODUCTION

Graded Lie algebras or Lie superalgebras (*i.e.* algebras having commuting and anti-commuting generators) were at one time considered as models for a more complete unified electroweak theory [1] [2] [3] [4] [5] as well as Grand Unified Theories [6]. Such graded algebras had many attractive features such as including both vector and scalar bosons within the same theory, fixing the Weinberg angle, and in some formulations the mass of the Higgs. However these graded algebras had shortcomings [7] [8] such as giving rise to negative kinetic energy terms for some of the gauge fields when the graded trace or supertrace was used.

In this paper we point out that this negative kinetic energy feature of the original graded algebras can be used to construct a model for phantom energy [9]. In addition to the phantom field there are other fields which arise in this model which could act as dark matter. The advantage of the combined phantom energy/dark matter model presented here is that it is derived from a modified gauge principle (*i.e.* the gauge principle applied to graded algebras) rather than being introduced phenomenologically. This feature fixes the parameters, such as the coupling between the phantom energy and dark matter, that are free in more phenomenological models.

Phantom energy is a form of dark energy which has a ratio of density to pressure less than -1 *i.e.* $w = p/\rho < -1$. Dark energy in general is a cosmological "fluid" with $w < -1/3$, which gives rise to an accelerated cosmological expansion. Dark energy was proposed to explain the accelerated expansion that was observed in studies of distance type Ia supernova [10] [11] [12]. There are various proposals as to the nature of dark energy which include a small, positive cosmological constant, quintessence models [13], brane world models [14] [15], Chaplyin gas [16], k-essence [17], axionic tensor fields [18] and others. A good review of the the current hypotheses as to the nature of dark energy and a guide to the literature can be found in reference [19]. Phantom energy is simply an extreme form of dark energy. The simplest model for phantom energy involves a scalar field with a negative kinetic energy term [9]

$$\mathcal{L}_p = -\frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - V(\phi) \tag{1}$$

The negative sign in front of the kinetic energy term makes this an unusual field theory. Such theories with negative kinetic energies have been investigated theoretically starting with [20] [21] where they were used to study wormhole solutions. Other papers considering scalar fields with negative kinetic energies can be found in [22] [23] [24] [25].

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However these theoretical studies did not garner much attention because of various problems with negative kinetic energies. Quantum mechanically such a field theories violate either conservation of probability or it has no stable vacuum state due to an energy density that is unboundedly negative. Nevertheless such unusual field theories are not absolutely ruled out [9], but one can place significant constraints on them [26]. Despite the theoretical problems of a scalar field with a negative kinetic energy term the reason to consider such a strange field theory is that recent observations give $-1.48 < w < -0.72$ [27] [28] [29] [30] [31] and thus favor $w < -1$. A very recent comparison of data from various sources can be found in [32]

The result $w < -1$ coming from the Lagrangian in (1) depends not only on the negative kinetic energy term, but also requires that the potential, $V(\phi)$, be present and satisfy some conditions. From (1) one can calculate p and ρ in the standard way as

$$\rho = T_{00} = -\frac{1}{2}\dot{\phi}^2 + V(\phi) \; ; \qquad\qquad p = -T_{ii} = -\frac{1}{2}\dot{\phi}^2 - V(\phi) \qquad (2)$$

where it is assumed that the scalar field is spatially homogeneous enough so that only the time variation is important. Using (2) to calculate w gives

$$w = \frac{p}{\rho} = \frac{-\frac{1}{2}\dot{\phi}^2 - V(\phi)}{-\frac{1}{2}\dot{\phi}^2 + V(\phi)} \qquad (3)$$

If $V(\phi) > 0$ and satisfies $\sqrt{2V(\phi)} > |\dot{\phi}|$ then one has $w < -1$. We will show that it is possible, using graded algebras, to construct a field theory that satisfies these conditions and so gives rises to phantom energy with $w < -1$. Unlike other models, the negative kinetic term comes from the structure of the graded algebras rather than being put in by hand. In addition there are other fields which could play the role of dark matter.

II.   SU(2/1) ALGEBRA

We briefly review the graded algebra SU(2/1), which we will use to construct the phantom energy model in the next section. The basic idea of using graded algebras to give phantom energy works for larger graded algebras like SU(N/1) with $N > 2$. We have taken SU(2/1) for simplicity.

We use the representation for SU(2/1) which consists of the following eight 3×3

$$EVEN: \quad T_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_2 = \frac{1}{2}\begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad T_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T_8 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

$$ODD: \quad T_4 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad T_5 = \frac{1}{2}\begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \quad T_6 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad T_7 = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}.$$

Except for T_8 this is the standard, fundamental representation of SU(3). The matrices on the first line above (*i.e.* T_1, T_2, T_3, T_8) are the even generators, and those on the second line (*i.e.* T_4, T_5, T_6, T_7) are odd generators. The even generators satisfy commutation relationships among themselves which can be written symbolically as $[EVEN, EVEN] = EVEN$. Mixtures of even and odd generators satisfy commutators of the form $[EVEN, ODD] = ODD$. Finally the odd generators satisfy anti-commutation relationships of the form $\{ODD, ODD\} = EVEN$. The further details of the SU(2/1) graded algebra can be found in the paper by Dondi and Jarvis [1] or in Ecclestone [7] [8]. The odd generators above are different than those usually taken in the literature. The connection of the odd generators above with those in [1] is given by $\bar{Q}^1, Q_1 = T_4 \pm iT_5$ and $\bar{Q}^2, Q_2 = T_6 \pm iT_7$. In the rest of the article we will use the convention that generators with indices from the middle of the alphabet (i, j, k) are the even generators, T_1, T_2, T_3, T_8, while indices from the beginning of the alphabet (a, b, c) are the odd generators T_4, T_5, T_6, T_7.

For the graded algebra one replaces the concept of the trace by the supertrace. For SU(2/1) this means that one writes some general element of the group as

$$M = \begin{pmatrix} A_{2\times 2} & B_{2\times 1} \\ C_{1\times 2} & d_{1\times 1} \end{pmatrix}.$$

The subscripts indicate the size of the sub-matrix. The supertrace is now defined as

$$\text{str}(M) = \text{tr}[A] - \text{tr}[d] \qquad (4)$$

which differs from the regular trace due to the minus sign in front of d.

In the next section we will need the supertraces of the various products of the eight generators (T_i, T_a), thus we collect these results here. For products of even generators we have

$$\text{str}(T_i T_j) = \delta_{ij}\frac{1}{2} \quad \text{except} \quad \text{str}(T_8 T_8) = -\frac{1}{2} \tag{5}$$

for the odd generators we have

$$\text{str}(T_4 T_5) = -\text{str}(T_5 T_4) = \frac{i}{2}, \quad \text{str}(T_6 T_7) = -\text{str}(T_7 T_6) = \frac{i}{2} \tag{6}$$

All other supertraces of the product of two matrices are zero.

III. PHANTOM ENERGY FROM SU(2/1)

In [1] vector fields were associated with the even generators and scalar fields with the odd generators as

$$A_\mu = igA_\mu^i T_i^{even} \;\; , \quad \phi = -g\varphi^a T_a^{odd} \tag{7}$$

The fields A_μ^i are regular commuting fields while φ^a are Grassmann fields. In block form one can write (7) as

$$A_M = \begin{pmatrix} A_\mu^3 + A_\mu^8 & A_\mu^1 - iA_\mu^2 & \varphi^4 - i\varphi^5 \\ A_\mu^1 + iA_\mu^2 & -A_\mu^3 + A_\mu^8 & \varphi^6 - i\varphi^7 \\ \varphi^4 + i\varphi^5 & \varphi^6 + i\varphi^7 & 2A_\mu^8 \end{pmatrix}.$$

In this fashion, *and by using the regular trace*, Dondi and Jarvis [1] showed that the Lagrangian

$$\mathcal{L} = \frac{1}{2g^2}\text{tr}(F_{MN}F^{MN}), \quad F_{MN} = \partial_M A_N - \partial_N A_M + [A_M, A_N], \tag{8}$$

reduced to an $SU(2) \times U(1)$ Yang-Mills Lagrangian for A_μ and a Higgs-like Lagrangian for ϕ. In (8) we use a different overall sign for the Lagrangian as compared to [1]. This comes because we have chosen different factors of i in the vector potentials defined below in (9). This was a more unified electroweak theory since $SU(2/1)$ is simple so that the Weinberg angle was fixed rather than being a parameter. However, if in (8) one used the correct SU(2/1) invariant supertrace then the Yang-Mills part of the reduced Lagrangian would have the wrong sign for the kinetic term for the $U(1)$ gauge field and the kinetic energy term for the scalar field would be lost. In [7] [8] this was used to develop arguments against the use of SU(2/1) as a unified electroweak theory.

Here we would like to use these apparent negative features of the graded algebras to construct a model for phantom energy. Instead of making the association between even/odd generators and vector/scalar fields made in (7) we will make the opposite choice

$$A_\mu = igA_\mu^a T_a^{odd} \;\; , \quad \phi = -g\varphi^i T_i^{even} \tag{9}$$

Because of the reversal of roles relative to (7) the fields A_μ^a are Grassmann fields while φ^i are regular, commuting fields. Then taking the correct SU(2/1) invariant supertrace we find that one of the scalar fields develops a negative kinetic energy term in addition to having a potential term which is positive definite. Thus the graded algebra gives rise to a phantom field.

With the choice in (9) the Lagrangian in (8) reduces as follows

$$\mathcal{L} = \frac{1}{2g^2}\text{str}(F_{MN}F^{MN}) = \frac{1}{2g^2}\text{str}\left[\left(\partial_{[\mu}A_{\nu]} + [A_\mu, A_\nu]\right)^2\right] + \frac{1}{g^2}\text{str}\left[\left(\partial_\mu\phi + [A_\mu, \phi]\right)^2\right] \tag{10}$$

We have introduced the notation $\partial_{[\mu}A_{\nu]} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Note that in comparison to other works such as [1] and [7] [8] we have not introduced extra Grassmann coordinates, ζ^α in addition to the normal Minkowski coordinates x^μ. Thus in [1] and [7] [8] coordinates and indices ran over six values – four Minkowski and two Grassmann. The final result in (10) can be obtained from [1] by dropping the Grassmann coordinates.

We first focus on the scalar term in (10). Inserting ϕ and A_μ from (9) into the last term in (10) we find

$$\begin{aligned} \mathcal{L}_S &= \frac{1}{g^2}\text{str}\left[\left(\partial_\mu\phi + [A_\mu, \phi]\right)^2\right] \\ &= \text{str}\left[\left(\partial_\mu\varphi^8 T_8 + igA_\mu^a\varphi^8[T_a, T_8]\right)^2\right] + \text{str}\left[\left(\partial_\mu\varphi^i T_i + igA_\mu^a\varphi^i[T_a, T_i]\right)^2\right] \end{aligned} \tag{11}$$

We now show that the first term in (11) takes the form of a phantom energy field. Expanding the first term in (11) gives

$$
\begin{aligned}
\mathcal{L}_{Ph} &= \mathrm{str}\left[\left(\partial_\mu\varphi^8 T_8 + igA_\mu^4\varphi^8[T_4,T_8] + igA_\mu^5\varphi^8[T_5,T_8] + igA_\mu^6\varphi^8[T_6,T_8] + igA_\mu^7\varphi^8[T_7,T_8]\right)^2\right] \\
&= \mathrm{str}\left[\left(\partial_\mu\varphi^8 T_8 - gA_\mu^4\varphi^8 T_5/2 + gA_\mu^5\varphi^8 T_4/2 - gA_\mu^6\varphi^8 T_7/2 + gA_\mu^7\varphi^8 T_6/2\right)^2\right]
\end{aligned}
\tag{12}
$$

We have used the representation of the SU(2/1) matrices from the previous section to evaluate the commutators. Using the supertrace results from (5) and (6) the expression in (12) yields

$$
\begin{aligned}
\mathcal{L}_{Ph} &= -\frac{1}{2}(\partial_\mu\varphi^8)^2 + \frac{1}{4}g^2(\varphi^8)^2\mathrm{str}\left[\left(A_\mu^4 T_5 - A_\mu^5 T_4 + A_\mu^6 T_7 - A_\mu^7 T_6\right)^2\right] \\
&= -\frac{1}{2}(\partial_\mu\varphi^8)^2 - \frac{i}{8}g^2(\varphi^8)^2\left(A_\mu^5 A^{4\mu} - A_\mu^4 A^{5\mu} + A_\mu^7 A^{6\mu} - A_\mu^6 A^{7\mu}\right) \\
&= -\frac{1}{2}(\partial_\mu\varphi^8)^2 - \frac{1}{16}g^2(\varphi^8)^2\left(A_\mu^+ A^{-\mu} - A_\mu^- A^{+\mu} + B_\mu^+ B^{-\mu} - B_\mu^- B^{+\mu}\right)
\end{aligned}
\tag{13}
$$

with $A_\mu^\pm = A_\mu^4 \pm iA_\mu^5$ and $B_\mu^\pm = A_\mu^6 \pm iA_\mu^7$. Both $A_\mu^\pm$ and $B_\mu^\pm$ are Grassmann so the last line in (13) can be written

$$
\mathcal{L}_{Ph} = -\frac{1}{2}(\partial_\mu\varphi^8)^2 - \frac{1}{8}g^2(\varphi^8)^2\left(A_\mu^+ A^{-\mu} + B_\mu^+ B^{-\mu}\right)
\tag{14}
$$

This is of the form of the phantom energy Lagrangian in (1) but with the potential involving not only the scalar field, φ^8, but Grassmann vector fields, $A_\mu^\pm$ and $B_\mu^\pm$. We will discuss these shortly. The minus sign in front of the kinetic energy term comes from taking the SU(2/1) invariant supertrace rather than the ordinary trace (see the second supertrace result in (5)).

We next focus on the other scalar fields, φ^i, $i = 1, 2, 3$ which come from the second term in (11). The calculation proceeds as in equations (12) - (13) but with φ^8 replaced by φ^i, $i = 1, 2, 3$. For example for φ^1 (12) becomes

$$
\mathcal{L}_{\varphi^1} = \mathrm{str}\left[\left(\partial_\mu\varphi^1 T_1 + gA_\mu^4\varphi^1 T_7/2 - gA_\mu^5\varphi^1 T_6/2 - gA_\mu^6\varphi^1 T_5/2 + gA_\mu^7\varphi^1 T_4/2\right)^2\right]
\tag{15}
$$

and (13) becomes

$$
\begin{aligned}
\mathcal{L}_{\varphi^1} &= \frac{1}{2}(\partial_\mu\varphi^1)^2 - \frac{1}{16}g^2(\varphi^1)^2\left(A_\mu^+ A^{-\mu} - A_\mu^- A^{+\mu} + B_\mu^+ B^{-\mu} - B_\mu^- B^{+\mu}\right) \\
&= \frac{1}{2}(\partial_\mu\varphi^1)^2 - \frac{1}{8}g^2(\varphi^1)^2\left(A_\mu^+ A^{-\mu} + B_\mu^+ B^{-\mu}\right)
\end{aligned}
\tag{16}
$$

There are two keys points: the kinetic term for φ^1 is positive since $\mathrm{str}(T_1 T_1) = +1/2$, and the potential term is the same as for φ^8. The other two even scalar fields follow the same pattern so that in total one can write

$$
\mathcal{L}_{DM} = \frac{1}{2}(\partial_\mu\varphi^i)^2 - \frac{1}{8}g^2(\varphi^i)^2\left(A_\mu^+ A^{-\mu} + B_\mu^+ B^{-\mu}\right)
\tag{17}
$$

where i is summed from 1 to 3. Thus the total scalar field Lagrangian resulting from (11) is the sum of (14) and (17). The scalar field in (14) has the "wrong" sign for the kinetic term and acts as a phantom field. The scalar fields in (17) are ordinary scalar field which we will interpret as a dark matter candidate. The phantom field and dark matter fields are coupled through the $A_\mu^\pm$ and $B_\mu^\pm$ fields. Thus our model provides a coupling between phantom energy and dark matter. Other models have been considered [33] where there is coupling between dark/phantom energy and dark matter.

We will now examine the Grassmann vector fields, $A_\mu^4, A_\mu^5, A_\mu^6, A_\mu^7$. The final Lagrangian for these fields will have a nonlinear interaction between the $A_\mu^\pm$ and $B_\mu^\pm$ fields. In analogy with QCD we argue that these fields form permanently confined condensates like $\langle A_\mu^4 A_\mu^5\rangle$ or $\langle A_\mu^4 A_\mu^-\rangle$. These then supply potential (mass-like) terms for the phantom energy and scalar fields of (14) and (17). This also avoids violation of the spin-statistics theorem since these condensates have bosonic statistics (they are composed of two Grassmann fields) and integer spin (they are composed of two integer spin fields). Having a potential term is crucial for the interpretation of φ^8 as a phantom energy field, since for a massless, non-interacting scalar field reversing the sign of the kinetic energy term does not lead a phantom field with $w < -1$ as can be seen from (3) if $V(\phi) = 0$. From (10) the vector part of the Lagrangian can be expanded as

$$
\begin{aligned}
\mathcal{L}_V &= \frac{1}{2g^2}\mathrm{str}\left[\left(\partial_{[\mu}A_{\nu]} + [A_\mu, A_\nu]\right)^2\right] = -\frac{1}{2}\mathrm{str}\left[\left(\partial_{[\mu}A_{\nu]}^a T_a + igA_\mu^a A_\nu^b\{T_a, T_b\}\right)^2\right] \\
&= -\frac{1}{2}\mathrm{str}\left[\left(\partial_{[\mu}A_{\nu]}^a T_a\right)^2\right] + \frac{g^2}{2}\mathrm{str}\left[\left(A_\mu^a A_\nu^b\{T_a, T_b\}\right)^2\right] = \mathcal{L}_{V1} + \mathcal{L}_{V2}
\end{aligned}
\tag{18}
$$

The commutator has become an anticommutator due to the Grassmann nature of the A_μ^a's. Also note that there is no cubic cross term between the derivative and anticommutator part. This comes about since the anticommutator, $\{T_a, T_b\}$ results in even generators, and the supertrace between odd and even generators vanishes. $\mathcal{L}_{V1}$ is a kinetic term for the fields and $\mathcal{L}_{V2}$ a potential term. We will now consider each of these in turn.

The kinetic part can be written explicitly as

$$\mathcal{L}_{V1} = -\frac{1}{2}\text{str}\left[\left(\partial_{[\mu}A_{\nu]}^4 T_4 + \partial_{[\mu}A_{\nu]}^5 T_5 + \partial_{[\mu}A_{\nu]}^6 T_6 + \partial_{[\mu}A_{\nu]}^7 T_7\right)^2\right] \tag{19}$$

Due to the property of the supertrace of the odd generators given in (6) it is only the cross terms between T_4, T_5 and T_6, T_7 which survive.

$$\mathcal{L}_{V1} = -\frac{i}{2}\left(\partial_{[\mu}A_{\nu]}^4 \partial_{[\mu}A_{\nu]}^5 + \partial_{[\mu}A_{\nu]}^6 \partial_{[\mu}A_{\nu]}^7\right) = -\frac{1}{4}\left(\partial_{[\mu}A_{\nu]}^- \partial_{[\mu}A_{\nu]}^+ + \partial_{[\mu}B_{\nu]}^- \partial_{[\mu}B_{\nu]}^+\right) \tag{20}$$

where we have used the anticommutating properties of the A_μ^a's. In the last step we have replaced the A_μ^a by $A_\mu^\pm$ and $B_\mu^\pm$. This kinetic part is reminiscent of the kinetic terms for a charged (i.e. complex) vector field.

Next we work out the form of the interaction terms coming from $\mathcal{L}_{V2}$. We do this explicitly for A_μ^4; the results for the other vectors fields can be obtained in a similar manner. The $A_\mu^a = A_\mu^4$ part of $\mathcal{L}_{V2}$ expands like

$$\mathcal{L}_{V2} = \frac{g^2}{2}\text{str}\left[\left(A_\mu^4 A_\nu^4 \{T_4, T_4\} + A_\mu^4 A_\nu^5 \{T_4, T_5\} + A_\mu^4 A_\nu^6 \{T_4, T_6\} + A_\mu^4 A_\nu^7 \{T_4, T_7\}\right)^2\right] \tag{21}$$

Using the explicit representations of the odd matrices we have $\{T_4, T_4\} = (T_3 + T_8)/2$, $\{T_4, T_5\} = 0$, $\{T_4, T_6\} = T_1/2$, $\{T_4, T_7\} = -T_2/2$. Squaring and using the supertrace results of (5) one finds that (21) becomes

$$\mathcal{L}_{V2} = \frac{g^2}{16}\left(A_\mu^4 A_\nu^6 A^{4\mu} A^{6\nu} + A_\mu^4 A_\nu^7 A^{4\mu} A^{7\nu}\right) \tag{22}$$

Note that there is no quartic term in A_μ^4 since the contributions from T_3 and T_8 cancel. The contribution from A_μ^5 looks the same as (22) but with $A_\mu^4 \to A_\mu^5$. The A_μ^6 and A_μ^7 terms can be obtained by making the exchange $A_\mu^4 \leftrightarrow A_\mu^6$ and $A_\mu^5 \leftrightarrow A_\mu^7$. Using the Grassmann character of the A_μ^a's one can see that the A_μ^4 and A_μ^6 contributions, and also the A_μ^5 and A_μ^7 contributions are the same. In total the interaction part of the vector Lagrangian can be written as

$$\begin{aligned}\mathcal{L}_{V2} &= \frac{g^2}{8}\left(A_\mu^4 A_\nu^6 A^{4\mu} A^{6\nu} + A_\mu^4 A_\nu^7 A^{4\mu} A^{7\nu} + A_\mu^5 A_\nu^6 A^{5\mu} A^{6\nu} + A_\mu^5 A_\nu^7 A^{5\mu} A^{7\nu}\right) \\ &= \frac{g^2}{16}\left(A_\mu^+ B_\nu^+ A^{-\mu} B^{-\nu} + A_\mu^+ B_\mu^- A^{-\nu} B^{+\nu}\right)\end{aligned} \tag{23}$$

In the last line we have written the interaction in terms of $A_\mu^\pm$, $B_\mu^\pm$.

The total Lagrangian for the vector Grassmann fields is, $\mathcal{L}_{V1} + \mathcal{L}_{V2}$, where $\mathcal{L}_{V1}$ is a kinetic term and and $\mathcal{L}_{V2}$ gives a nonlinear interaction term between $A_\mu^\pm$ and $B_\mu^\pm$. We assume that the interaction is strong enough that the fields, $A_\mu^\pm$ and $B_\mu^\pm$ are permanently confined into condensates

$$\langle A_\mu^+ A^{-\mu}\rangle = \langle B_\mu^+ B^{-\mu}\rangle = v \tag{24}$$

Given the symmetry between the $A_\mu^\pm$ and $B_\mu^\pm$ fields we have taken them to have the same vacuum expectation value. This conjectured condensation is similar to the *gauge variant*, mass dimension 2 condensate, in regular Yang-Mills theory, $\langle \mathcal{A}_\mu^a \mathcal{A}^{a\mu}\rangle$. Despite being *gauge variant* this quantity has been shown [34] [35] [36] [37] to have real physical consequences in QCD. Here $\mathcal{A}_\mu^a$ is a normal SU(N) Yang-Mills field. In [38] a BRST-invariant mass dimension 2 condensate was constructed which was a combination of the quadratic gauge field term – $\langle \mathcal{A}_\mu^a \mathcal{A}^{a\mu}\rangle$ – plus a quadratic Fadeev-Popov [39] ghost field term – $i\alpha\langle \mathcal{C}^a \bar{\mathcal{C}}^a\rangle$ – where α was a gauge parameter. In the Landau gauge, $\alpha = 0$, this reduced to a pure quadratic gauge field condensate $\langle \mathcal{A}_\mu^a \mathcal{A}^{a\mu}\rangle$. Note that the ghost fields, $\mathcal{C}^a, \bar{\mathcal{C}}^a$, are bosonic (i.e. scalar) Grassman fields. This mass dimension 2 condensate gives the gluon a mass [40] [41] [42]. Estimates have been made for $\sqrt{\langle \mathcal{A}_\mu^a \mathcal{A}^{a\mu}\rangle}$ using lattice methods [34] [35] [42], analytical techniques [43] or some mixture. All these various methods give a condensate value in the range $\sqrt{\langle \mathcal{A}_\mu^a \mathcal{A}^{a\mu}\rangle} \approx 1$ GeV. Given the similarities between the regular gauge field condensate of [34] [35] [36] [37] [38] and that on the left hand side of (24) we can estimate the vacuum expectation value as $v \approx 1$ GeV ^{2}.

Now inserting these vacuum expectation values from (24) into (14) yields

$$\mathcal{L}_{Ph} = -\frac{1}{2}(\partial_\mu \varphi^8)^2 - \frac{v}{4}g^2(\varphi^8)^2 \tag{25}$$

This is of the form (1) with $V(\varphi^8) = \frac{v}{4}g^2(\varphi^8)^2$. This will give phantom energy with $w < -1$ if $\frac{g}{2}|\varphi^8|\sqrt{2v} > |\dot{\varphi}^8|$. If the vacuum expectation value, v, change over time it is possible to cross into (out of) the phantom regime if v increases (decreases). Thus whether one has phantom energy or not would depend on the dynamical evolution of v. Such models, where one crosses the "phantom divide", have been considered in [44] [45] [46] [47]. Usually in such models it is the sign in front of the kinetic energy term that is modified, whereas in the present case it is a modification of the potential which causes the transition between phantom and non-phantom phases. Further extensions of these "quintom" models can be found in [48] [49] [50] [51] [52].

Inserting the vacuum expectation values into the Lagrangian for the scalar fields $\varphi_1, \varphi_2, \varphi_3$, equation (17) becomes

$$\mathcal{L}_{DM} = \frac{1}{2}(\partial_\mu \varphi^i)^2 - \frac{v}{4}g^2(\varphi^i)^2 \tag{26}$$

The Lagrangian for these fields is for a standard, non-interacting scalar with mass $m = \frac{g}{2}\sqrt{2v}$. These massive scalar fields could be taken as a candidate for cold dark matter if m (i.e. v) is chosen appropriately. For example, using the similarity between the condensate of (24) and the mass dimension condensate of [34] [35] [36] [37] [38] one might set $v \approx 1$ GeV ^{2}. This would given $m \approx 1$ GeV making φ^a a cold dark matter candidate.

The original Lagrangian we started with in (10) has no coupling to the usual Standard Model fields except through gravity. This would certainly explain why these phantom energy and dark matter fields have not been seen since they could only be detected through their gravitational influence. However if this is the path nature chooses it would be hard if not impossible to get any kind of experimental signal of these phantom energy/dark matter candidates. One could introduce some effective coupling between the phantom energy/dark matter fields of (10) and the usual Standard Model fields. More rigorously one might try to use some larger SU(N/1) group, but having some of the vector fields be associated with the even generators and some associated with the odd generators and similarly for the scalar fields. In this way it might be possible to have a new kind of "Grand Unified Theory": from a single Lagrangian one could have Standard Model gauge fields as well as new fields that would be phantom energy and dark matter candidates, instead of extra Grand Unified gauge bosons.

The Grassmann vector fields are an odd feature of this model since they would violate the spin-statistics theorem. These Grassmann vector fields are similar to the Fadeev-Popov ghosts [39] which are scalar fields with Fermi-Dirac statistics. The Fadeev-Popov ghosts however are not real particles in that they never appear as asymptotic states. In order to avoid having the Grassmann vector fields violate the spin-statistics theorem, we have postulated that the composite states, $A_\mu^+ A^{-\mu}$ and $B_\mu^+ B^{-\mu}$ are permanently confined so that the particles associated with $A_\mu^\pm$ and $B_\mu^\pm$ never appear as asymptotic states. Since the composites are ordinary fields (integer spin with bosonic statistics) violation of the spin-statistics theorem is avoided. These vectors fields act as a second dark matter component in addition to the three scalar fields φ_i. There have been other recent proposals for dark matter candidates with non-standard relationships between spin and mass dimension. In [53] [54] a spin 1/2 dark matter candidate was proposed which has mass dimension 1. In the present case our vector fields, $A_\mu^\pm$, $B_\mu^\pm$, have the same mass dimension – 1 – and statistics – fermionic – as the dark matter candidate in [53] [54], and only differ in the value of spin – 1 versus 1/2.

IV. SUMMARY AND CONCLUSIONS

We have given a model for phantom energy using a modification of the graded Lie algebras models which attempted to give a more unified electroweak theory, or Grand Unified theories. Despite interesting features of the original graded Lie algebra models (*e.g.* prediction of the Weinberg angle and having both vectors and scalars coming from the same Lagrangian) they had shortcomings. Chief among these was that if one used the correct SU(N/1) invariant supertrace then some of the vector fields had the wrong sign for the kinetic energy term in the Lagrangian. In the original models the vector fields were associated with the even generators of the algebra and the scalars fields were associated with the odd generators. Here we took the reverse identification (scalar field → even generators and vector field → odd generators) which led to the wrong sign kinetic energy term coming from a scalar field rather than from a vector field. The wrong sign scalar field, φ^8, gives a model of phantom energy, while the other scalar fields, φ^i, and the vector fields, A_μ^a, act as dark matter components. In the way our model is formulated here all the fields are truly dark in that they have no coupling to any of the Standard Model fields and would thus only be detectable via their gravitational interaction. This would make the experimental detection of these dark fields impossible through non-gravitational interactions. However the above is intended only as a toy model of how a phantom energy field can emerge naturally

from a gauge theory with a graded Lie algebra. A more experimentally testable variation of the above toy model could have some coupling between the scalar and vector fields of the present model and the Standard Model fields. Such a coupling could be introduced in a phenomenological fashion via some *ad hoc* coupling. A more interesting option would be to consider some larger graded algebra, such as SU(N/1). Some of the fields could be given the standard assignment of even or odd generators (*i.e.* as in (7)) while others could be given the assignment in (9). The fields given the standard assignment would give standard gauge fields, while fields given the non-standard assignment would give phantom energy and dark matter fields. This would give a new type of "Grand Unified Theory" with the phantom energy and dark matter fields replacing the extra gauge bosons of ordinary Grand Unified Theories. Other authors [55] have used non-standard gauge groups such as SO(1,1) to give models of phantom energy.

An important feature of the above model is the assumption that the Grassmann vector fields form permanently confined condensates. This was a crucial to our phantom energy model since it leads to a condensate of the $A_\mu^\pm$ and $B_\mu^\pm$ fields. This in turn gave a potential $V(\varphi^8) = \frac{v}{4}g^2(\varphi^8)^2$ for the φ^8 field which was of the correct form to allow φ^8 to act as phantom energy. Aside from the present application to phantom energy one might try to use the above mechanism to generate standard symmetry break by starting with a graded Lie algebra but using all vector fields rather than mixing vector and scalar. In this way some of the vector fields would be standard vector fields, while other would be Grassmann vector fields. By the above mechanism the Grassmann vector fields would form condensates which would then give masses to the standard vector fields *i.e.* one would have a Higgs mechanism with only vector fields).

One additional avenue for future investigation is to see if one could have a phantom energy model with the original graded Lie algebra models (*i.e.* with vector fields assigned to even generators and scalars to odd) but using the supertrace. One would then have the problem of some of the vector fields having the wrong sign in the kinetic term, but this might then give a phantom energy model with a vector rather than scalar field.

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